Identification of naturally fractured reservoirs by optimal control methods*

Anatol Winter

Winter, A.: Identification of naturally fractured reservoirs by optimal control methods. *Danm. geol. Unders., Årbog 1980:* 75–89. København, 15. december 1981.

The dynamic behavior of naturally fractured petroleum reservoirs is described by a mathematical model based on the assumption that the reservoir rock can be represented by an array of identical, rectangular parallelopipeds where the blocks correspond to the matrix and the spacing between, to the fractures. Material balance conditions imposed on the oil and/or gas phases result, in general, in a coupled set of nonlinear partial differential equations. By solving this system of equations one gets reservoir pressure history. Physical properties of the reservoir such as porosity and permeability and those characterizing the deviation of the behavior of a medium with »double porosity« from that of a homogeneous porous medium are represented by parameters appearing in the model. This paper deals with the problem of determination of the parameters described above in real life naturally fractured reservoirs.

Anatol Winter, Geological Survey of Denmark, Thoravej 31, DK-2400 Copenhagen NV, Denmark.

In this paper the problem of estimation of naturally fractured petroleum reservoir properties on the basis of data obtained during a production test is described. More specifically, the paper deals with interference tests performed in hard formations where presence of fractures is of great importance: they act as channels between matrix and a borehole. On the other hand, their contribution to the overall porosity and their volume are negligible as compared to the matrix porosity and the total volume of the reservoir.

In an interference test one deals with a system composed of two (or more) wells: an active well (producing or injecting fluids) and an observation well. The active well is shut in after some time and the resulting pressure change is registered at the observation well. The pressure response can help determine formation continuity, degree of fracturing, and areal average transmissibility and storage between a well pair.

The problem of estimation of reservoir parameters using production data is inherently undetermined because the number of parameters usually exceeds the available data. The undeterminacy can be reduced by a classification of porous rock property distributions, each possible distribution being referred

* This paper was presented at Symposium "Mathematical Models and Simulation" held as part of the 26th International Geological Congress, Paris, France, July 1980.

to one of a final number of categories. The success of this approach (also known as parametrization) requires that the number of potential categories is small.

A typical example of parametrization is the zonation technique: the estimated property distributions are assumed to be uniform within each of several regions of the reservoir (zones). Thus, they change abruptly at the boundaries of the zones causing a considerable modelling error.

An alternative approach to dimensionality reduction is that based on Bayesian estimation where an a priori statistical information concerning the unknowns is incorporated in the estimation algorithm.

Recently, several papers have been published reflecting increasing interest in methods used to detect fractures from logs (e.g., Aguilera 1976). Warren & Root (1963) pointed out that for good models of naturally fractured reservoirs the most important aspect of the model design strategy is that "all available measurements and observations are utilized, furthermore the model must be consistent with the physical inferences obtained from the performance of actual reservoirs of this particular type".

The parallel problem of estimation of properties in a homogeneous petroleum reservoir has been treated previously by Gavalas et. al. (Gavalas 1976).

Some remarks on the origin of naturally fractured reservoirs

The genesis of naturally fractured reservoirs, in which salt domes are part of a trapping system, can be conveniently explained via the fluid mechanical hypothesis for the formation of salt domes. The first to demonstrate the applicability of this hypothesis to the formation of salt domes was Nettleton (1936). He assumed that the density difference between the salt and the surrounding sediments acts as an upward driving force of buoyancy. Moreover, the salt and the overlying sediments behave like highly viscous fluids. Since the salt is covered by denser sedimentary strata, the system becomes inherently unstable and any initial perturbation (caused, for example, by some tectonic movement) will start the flow of the salt from an underlying bed to a rounded salt pillow. The next steps in this dynamic process are, broadly speaking, as follows (see Braunstein et. al. 1968):

a. the flow continues into the centre of the pillow, doming the overlying strata,

b. simultaneously with the process described in a, the area from which salt has flowed subsides,

c. the strata overlying the flowing salt are exposed to tension which causes a development of fractures.

The problem of estimation of parameters concerned with the fractured reservoir rock will be taken up in the subsequent sections.

Mathematical model of a single-phase homogeneously fractured reservoir

A petroleum reservoir can be viewed as a gigantic chemical reactor and, consequently, it is modelled according to the same principles as chemical reactions in a spatial domain. More specifically, the following classification can be used as a frame for all models (Arnold 1980):

i) global description (no diffusion, spatially homogeneous or "well stirred" case) versus local description (with diffusion, spatially inhomogeneous case),

ii) deterministic description (macroscopic, in terms of concentrations) versus stochastic description (operating with the number of particles and including internal fluctuations).

By a combination of i) and ii) one gets four essential mathematical models. In this paper we shall be concerned only with the local-deterministic type. The reader interested in more details about various reaction models may consult Prigogine et. al. (1977).

The problem of modelling a behaviour of naturally fractured reservoirs has been treated in many papers. The classical work dealing with this subject is that of Warren and Root (1963). In their model the fractured reservoir is represented by a system of identical, rectangular parallelopipeds separated by a regular network of fractures (see Fig. 1). Moreover, the formation fluid is assumed to flow through these (high conductivity) fractures. The crucial assumption i the Warren-Root model is that each fracture is parallel to one of the principal axis of permeability. A somewhat different model was suggested by Odeh (1965). He makes no assumptions about the size of the matrix blocks, their uniformity or geometric pattern. The only extension of the Odeh's model as compared with the conventional nonfractured reservoir is an introduction of the parameter β describing the degree of fracturing meant as fractures' bulk volume per unit reservoir volume. Finally, Kazemi (1969) described a naturally fractured reservoir using a multilayered system com-

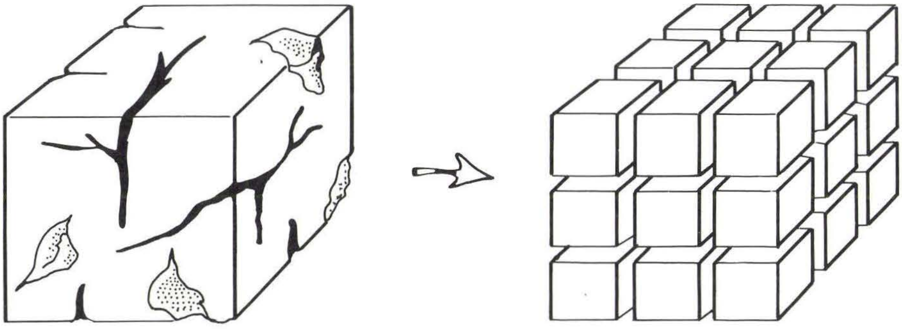

Fig. 1. Mathematical idealization of the naturally fractured reservoir.

posed of a thin, highly conductive layer representing the fracture which is adjacent to a thicker layer endowed with low conductivity and high storage capacity representing the matrix (see Fig. 2). The results given by Kazemi are consistent with those of Warren- Root and Odeh. For more details and comparative analysis of different models the reader may consult the survey paper of Crawford et. al. (Crawford 1976).

The basic differential equation governing the radial, single phase flow of oil in a naturally fractured reservoir is based on the following assumptions:

a. the reservoir rock corresponding to the primary porosity is contained within an array of blocks which act like sources feeding the fractures with oil,

b. all fluid flow is due to fractures (there is no fluid traffic in the primary blocks),

c. the reservoir is assumed to be homogeneously fractured, i.e., the fractures' flow capacity and the degree of fracturing in the reservoir are uniform,

d. the usual assumptions ensuring fully radial flow are fulfilled (see Dake 1978).

It should be noted that the Warren-Root model formally belongs to the zonation type of parametrization and, consequently, it has all the shortcomings of that approach: the constraint of uniformity of rock properties within each zone is very inflexible and does not usually correspond to geological knowledge about the reservoir. In real life petroleum reservoirs the estimated

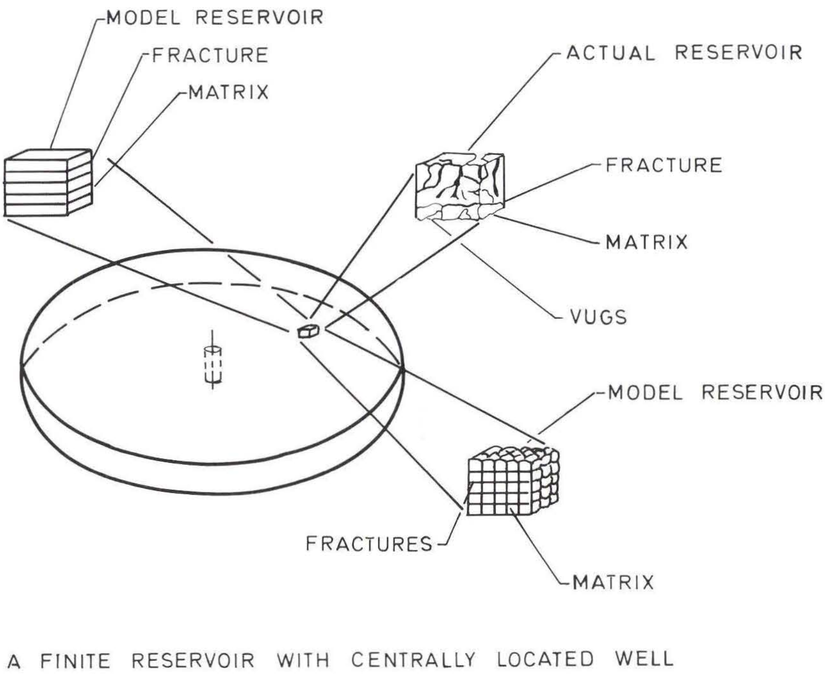

Fig. 2. Mathematical models of naturally factured reservoir [after Kazemi (1969)]

parameters cannot be described with sufficient accuracy by a piecewise constancy. Since they can be regarded as the result of several random conditions during sedimentation, a model given by a random process with a certain probability distribution seems to be much more appropriate.

Looking more closely at the Warren-Root model one can easily discover that it implies heterogeneity only on a microscopic scale. If the dimensions of the blocks are small in comparison with the dimensions of the reservoir, it may be considered as homogeneous. Thus, the "zones" in the model should be viewed as a tool in the process of averaging the unknowns rather than reflecting the existing physical spacing. In fact, it has been shown that the behaviour of a homogeneously heterogeneous system can be approximated by that of a homogeneous system with a (global) permeability equal to a geometric mean of the individual (local) permeabilities (see Matheron 1966).

In order to derive the differential equation for the fluid flow in a homogeneously heterogeneous system two pressures are defined at each point following Warren & Root (1963).

$$p_1 = \int_V p\, g_1(V)dV \,/\, \int_V g_1(V)dV, \tag{1}$$

$$p_2 = \int_V p\, g_2(V)dV \,/\, \int_V g_2(V)dV, \tag{2}$$

where V denotes an elementary volume and

$$g_1(V) = \begin{cases} 1, & \text{in the primary porosity,} \\ 0, & \text{outside of primary porosity,} \end{cases}$$

$$g_2(V) = \begin{cases} 1, & \text{in the secondary porosity,} \\ 0, & \text{outside of secondary porosity,} \end{cases}$$

and two porosities

$$\int_V g_1(V)dV = \varphi_1, \tag{3}$$

$$\int_V g_2(V)dV = \varphi_2, \tag{4}$$

We are now in a position to derive the partial differential equation for radial flow in naturally fractured reservoirs.

From the principle of mass conservation (Fig. 3)

Mass flow rate IN – Mass flow rate OUT = Rate of change of mass in the volume element

$$q\varrho_2|_{r+dr} - q\varrho_2|_r = 2\pi\, rhdr \left(\varphi_1 \frac{\partial p_1}{\partial t} + \varphi_2 \frac{\partial p_2}{\partial t}\right) \tag{5}$$

where $2\pi rhdr$ is the total volume of the infinitesimal element of thickness dr and

q=flow rate, positive for production and negative for injection,

ϱ_i=density of oil (i=1 refers to the matrix, i=2 to the fractures).

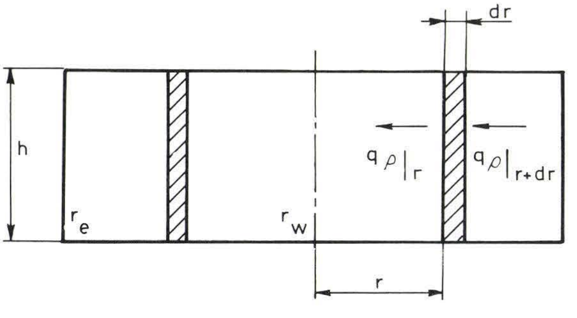

Fig. 3. Radial model of a single phase fluid flow

After some straightforward, but tedious calculations the following equations can be obtained (see Kazemi 1969).

$$\frac{\partial^2 p_{2D}}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial p_{2D}}{\partial r_D} - (1-\omega) \frac{\partial p_{1D}}{\partial t_D} = \omega \frac{\partial p_{2D}}{\partial t_D}$$

$$0< r_D<\infty, \qquad t_D> 0 \tag{6}$$

$$(1- \omega) \frac{\partial p_{1D}}{\partial t_D} = \lambda(p_{2D}-p_{1D}),$$

$$0< r_D<\infty, \qquad t_D> 0 \tag{7}$$

where the second equation (7) describes the rate of feed to the fractures by the matrix blocks and

r_D = dimensionless radius: r/r_w,

t_D = dimensionless time:
$2.637 \times 10^{-4} (k_2t)/((c_1\varphi_1+\varphi_2c_2)\mu r_w^2)$,

k_2 = effective permeability (md),

$p_D(r_D,t_D)$= dimensionless pressure: $(p_i-p)/(\frac{141.2\ q\mu\ B}{kh})$,

μ = viscosity(cp),

λ = interporosity flow parameter:

$$\frac{\alpha k_1 r_w^2}{k_2},$$

ω = ratio of storage capacity of fractures to the total storage capacity:

$$\frac{\varphi_2 c_2}{\varphi_1 c_1+\varphi_2 c_2}$$

The boundary conditions corresponding to the interference test are as follows

a) $p_{1D} = p_{2D} = 0,\ t_D=0,\ 0< r_D< \infty$ (8)

b) $\lim_{r_D\to 0} r_D \frac{\partial p_{2D}}{\partial r_D} = -1, \qquad t_D >0$

c) $$\lim_{r_D \to \infty} p_{2D} = 0 \quad , t_D > 0 \tag{10}$$

In the next section the identification problem will be formulated. We have assumed that the solution of eqs. (6) and (7) subject to the boundary conditions (8), (9) and (10) is available either in an analytical or in a numerical form.

Formulation of the identification problem

The usual procedure used in the identification of naturally fracture reservoir properties via pressure tests is, broadly speaking, as follows:

a) define a mathematical model describing fluid flow in the reservoir rock,

b) derive the solution of the equation introduced in (a). Variable production tests can be treated by convoluting the constant rate solutions. Problems with two or more wells with different production schedules can be solved by superposition.

c) match the theoretically predicted pressure response (obtained via the mathematical model) with measured field data and identify the parameters appearing in the model. If the unknown parameters can be described by an a priori probability distribution, it should be incorporated into the model. This is equivalent to the requirement that the parameters follow some preconceived pattern.

Among the most important sources of information about the estimated parameters the following can be mentioned:

a) well logs run inside the drilled wells,
b) analysis of rock and fluid samples,
c) production tests and history,
d) seismic data.

Since changes in lithology may have similar influence on recording as the presence of fractures, the interpretation of available information should be

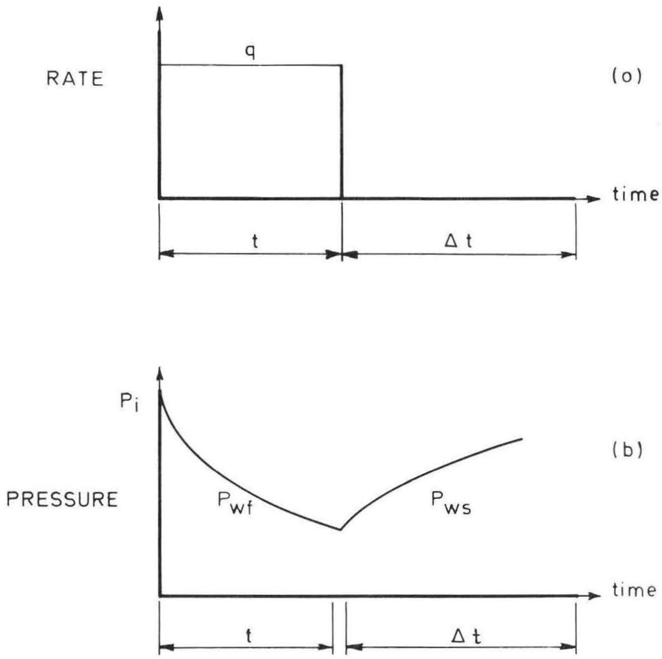

Fig. 4. Typical build-up test; (a) rate, (b) Wellbore pressure response.

done with care. In many cases the prior statistical information, concerning the unknowns, is limited to the knowledge of the size of their fluctuations.

The property estimation problem is defined as follows. Firstly, a conventional buildup test is matched to the Warren-Root model and the parameters,

$$\omega \triangleq \varphi_2 c_2/(\varphi_1 c_1+\varphi_2 c_2) \tag{11}$$

$$\lambda \triangleq (\alpha k_1/k_2) r_w^2$$

are determined. It should be noted that the second of the above parameters can be used to calculate fracture permeability k_2 (all other parameters assumed known from laboratory experiments and core studies).

Secondly, the average diffusivity between the producing and the observation wells

$$D_{av} \triangleq k/\varphi\mu c = (k_1+k_2)/((\varphi_1 c_{1tot}+ \varphi_2 c_{2tot})\mu) \tag{13}$$

where

$$c_{1tot}= c_o+c_w S_{wm}/(1\text{-}S_{wm}) + c_1/(1\text{-}S_{wm}) \tag{14}$$

$$c_{2tot}= c_o+c_2, \tag{15}$$

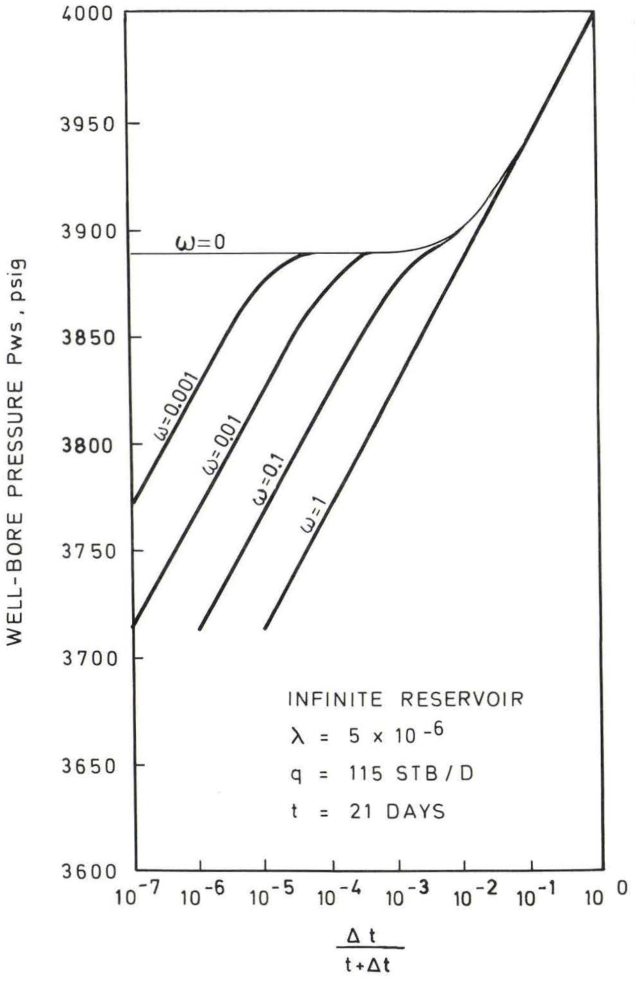

Fig. 5. Build-up curves obtained via Warren-Root model. (after Warren, 1963).

is estimated via an interference test. By combining the above equations one can estimate φ_2 and c_2. Especially, the parameter φ_2 (fracture porosity) is extremely important in reservoir engineering calculations. In som cases (when there is no porosity in the matrix blocks) it is not accessible for direct measurements and an interference test is the only way to estimate its range.

In a buildup test a well is flowed at a constant rate q for a total time t and then closed in. The rate schedule and the corresponding pressure response for a simple buildup test are shown in Fig. 4.

The parameter ω can be obtained from the following relationship

$$\omega = \text{antilog}\ (-\delta p/m) \tag{16}$$

where

δp=vertical separation of the two straight lines(psi) of the buildup plot,

m= slope of the abovementioned lines (see Fig. 5).

Recently, a method of estimation of the parameter λ via the coordinates of the inflection point on a buildup plot was presented (see Uldrich, 1979).

In the next section the main lines of the solution of the identification problem will be described.

Strategy of the solution

The interference test data are matched to the solution of the system of equations (6) and (7) subject to the boundary conditions (8), (9) and (10) by minimizing the following functional with respect to the parameters ω and λ:

$$J_p = \sum_{i=1}^{M} (w_i) \{ p_{wint}(\omega,\lambda, (\triangle t_i)_D) - (p_{wint})_{ob} (\triangle t_i)_D\}^2 \qquad (17)$$

where

M=total number of measurements,

$p_{wint}(\triangle t_i)_D$= wellbore pressure at the observation well computed at $(\triangle t_i)_D$ from the theoretical model,

$(p_{wint})_{ob}(\triangle t_i)_D$= measured wellbore pressure at the observation well,

w_i= weighting coefficients depending on statistics of different observation errors.

The minimization of J_p corresponds to the incorporation the information contained in the pressure measurements into the estimation procedure. In order to include also the prior geological information about the identified parameters we have to redefine the objective function (17)

$$J= J_p + J_g \qquad (18)$$

The term J_g is defined in the following way

$$J_g = \gamma^T P_o^{-1} \gamma \tag{19}$$

where

$\gamma = \{ (\omega - \hat{\omega}) (\lambda - \hat{\lambda}) \}$,

$P_o = E\{\gamma\gamma^T\}$ is the prior covariance associated with the parameter γ,

and $\hat{\omega}$, $\hat{\lambda}$ denote the prior mean values of the respective parameters.

In order to estimate the parameters ω and λ, we have to find a particular vector

$$\tau = (\omega^o, \lambda^o),$$

so that the composite functional J (defined by eq. 18) is minimum.

The above considerations can be generalized to cover a multiwell case. The simplest example is that with N wells spaced equidistantly along a straight line.

Let $\pi^T = (\omega^T, \lambda^T)$ be the 2N composite vector of parameteres, $e_1 > e_2 > \ldots > e_{2N}$ the characteristic values of P_o (P_o is a symmetric and positive definite matrix, consequently, all of its eigenvalues are real and positive), and $z(1), z(2), \ldots z(2N)$ the corresponding characteristic vectors. Defining the matrices

$$Z = (z(1), z(2), \ldots, z(2N))$$

$\Lambda = \mathrm{diag}(e_1, e_2, \ldots, e_{2N})$,

we can write the prior covariance matrix in the following form

$$P_o = Z \Lambda Z^T \tag{20}$$

Assuming that $\pi - \hat{\pi}$ is a stationary random vector, it can be decomposed along the complete, orthonormal set of vectors $z(1), z(2), \ldots, z(2N)$

$$\pi - \hat{\pi} = Zu = \sum_{j=i}^{2N} u_j z(j) \tag{21}$$

or (as a result of orthogonality of Z)

$$u = Z^T(\pi - \hat{\pi})$$

where $\hat{\pi} = E\{\pi\}$ and u is a Gaussian random vector with the following properties

$$E\{u\} = O$$

$$E\{uu^T\} = Z^T P_o Z = \Lambda \tag{22}$$

The above results can be used to rewrite the Bayesian penalty term in the eq.19. Using the fact that $Z^T P_o Z$ is diagonal (with entries $1/e_J$), and that $\gamma = \pi - \hat{\pi} = Zu$, we get

$$J_g = (\pi - \hat{\pi})^T P_o^{-1} (\pi - \hat{\pi}) = (Zu)^T P_o^{-1} (Zu) = u^T Z^T P_o^{-1} Zu = u^T \Lambda^{-1} u$$

Thus,

$$J_g = \sum_{j=1}^{2N} \frac{u_j^2}{e_j^2} \tag{23}$$

and the estimation problem has been reduced to the class of nonlinear least squares problems. It can be solved by, for example, the Newton-Raphson or the Gauss-Newton method (see Mc Keown 1979).

In many real life cases the reservoir properties are spatially correlated. Sometimes the best evidence of such correlation can be obtained by seismic facies analysis (see Payton, ed. 1977). Seismic facies analysis involves the description and geologic interpretation of seismic reflection parameters such as amplitude, frequency and interval velocity. Frequency can be related to lateral changes in interval velocity and, consequently, to variations in porosity.

In the case of spatial correlation of the estimated parameters the characteristic values decline with increasing j (see eq. 23) and those u_j which correspond to very small e_j will be effectively suppressed in the minimization. Thus, the modified functional J_g can be rewritten as follows

$$J_g = \sum_{j=1}^{K} \frac{u_j^2}{e_j} \tag{24}$$

The number of parameters to be estimated is now K ($u_1, \ldots u_K$) instead of 2N. This is an important contribution of seismic stratigraphy to the alleviation of the »curse of dimensionality". Unfortunately, practical applicability of this approach depends heavily on the quality of seismic data.

Concluding remarks

In this paper we have described a methodology for parameter identification in naturally fractured reservoirs. The estimation problem has been posed as a minimization of a composite functional including the prior geological information about the unknowns. Its solution will usually require the knowledge of the derivatives of the abovementioned functional with respect to the estimated parameters. An efficient calculation method using the adjoint equation approach of the optimal control theory has been given by Chavent (1975).

It should be noted that in practical applications of the described methodology some serious obstacles can be encountered. They are mainly due to a poor quality of production tests. Consequently, there will be no sufficient basis for the prior statistical information about the estimated parameters. Also, in some cases, certain combination of the parameters result in the pressure response which is identical for homogeneous and uniformly fractured cases (see Odeh 1965).

The approach to parameter identification described in this paper stresses a multidisciplinary character of a reservoir developement study: geological, geophysical and engineering knowledge must be combined in order to produce reliable estimates.

Notation

(only the parameters not defined explicitly in the paper)

φ_1 = matrix porosity, fraction
φ_2 = fracture porosity, fraction
c_o = oil compressibility, psi^{-1}
c_w = water compressibility, psi^{-1}
c_1 = matrix compressibility, psi^{-1}
c_2 = fracture compressibility, psi^{-1}
h = reservoir thickness, ft
S_{wm} = connate water saturation in the matrix, fraction
r_w = wellbore radius, ft, r_e = reservoir radius, ft
k_1 = matrix permeability, md
k_2 = fracture permeability, md
p_i = initial reservoir pressure, psi
α = shape factor depending on geometry of the matrix blocks, $1/ft^2$
B = formation volume factor, fraction

Subscripts

1 = primary porosity, 2 = secondary porosity
D = dimensionless, f=flowing (pressure), s = shut-in (pressure)
int = interference (refers to properties measured during an interference test)
w = wellbore.

References

Aguilera R. 1976: Analysis of naturally fractured reservoirs from conventional well logs.Journal of Petroleum Technology, (July): 764–772.

Arnold L. 1980: On the consistency of the mathematical models of chemical reactions. Procedings of the international symposium on synenergetics, Bielefeld, Fed. Rep. of Germany, September 24–29: 107–118.

Braunstein J. 1968: Diapirism and diapirs, a symposium.

Chavent G. et. al 1975: History matching by use of optimal control theory. Society of Petroleum Eng. Journal, 15: 74–86.

Crawford G. E. 1976: Analysis of pressure buildup tests in a naturally fractured reservoir. Journal of Petroleum Technology, (November): 1295–1300.

Dake L. P. 1978: Fundamentals of Reservoir Engineering, Elsevier.

Erlougher R. 1979: Advances in well test analysis. SPE monograph no 5.

Gavalas G. R. et. al. 1976: Reservoir history matching by use of optimal control theory. Society of Petroleum Engineers Journal, 16: 337–350.

Harris D. G. et. al. 1977: Synenergetism in reservoir management- the geologic perspective. Journal of Petroleum Technology, (July): 761–770.

Kazemi H. 1969: Pressure transient analysis of naturally fractured reservoirs with uniform fracture distribution. Society of Petroleum Engineers Journal, (December): 451–462.

Matheron G. 1966: Structure et composition des perméabilities. Revue de l'Institut Français du Petrole, (Avril): 564–580.

Matheron G. 1967: Composition des perméabilities en milieu poreux héterogene. Revue de l'Institut Français du Petrole, (Mars): 443–466.

Mc Keown J. J. 1979: Nonlinear least-squares problems. Lecture notes of the course on "Optimization Techniques and Applications", University of Bergamo, (September): 17–28.

Nettleton L. L. 1934: Fluid mechanics of salt domes. Bull. A.A.P.G. 18: 175–204.

Odeh A. S. 1965: Unsteady-state behaviour of naturally fractured reservoirs. Society of Petroleum Engineers Journal, (March): 60–66.

Payton C. (ed). 1977: Seismic stratigraphy-applications to hydrocarbon exploration, memoir no 26 of the A. A. P. G.

Prigogine I. et. al. 1977: Self-organization in non equilibrium systems, Wiley-Interscience.

Prigogine I. et. al. 1979: La nouvelle alliance. Gallimard.

Saidi A. M. et. al. 1979: Mathematical simulation of fractured reservoir performance based on physical model experiments. Proceedings of the X-th World Petroleum Congress.

Suau J. et. al. 1980: Fracture detection from logs, The Log Analytyst, (March-April): 3–13.

Uldrich D. O. 1979: A method for estimating the interporosity flow parameter in naturally fractured reservoir. Society of Petroleum Engineers Journal, (October): 324–332.

Warren J. E. et. al. 1961: Flow in heterogeneous porous media. Society of Petroleum Engineers Journal, (September): 153–169.

Warren J. E. et. al. 1963: The behaviour of naturally fractured reservoirs. Society of Petroleum Engineers Journal, (September): 245–265.